USDA FOREST SERVICE RESEARCH NOTE NE-187
1974

Northeastern Forest Experiment Station

FOREST SERVICE, U.S. DEPT. OF AGRICULTURE, 6816 MARKET STREET, UPPER DARBY, PA. 19082

ODC 646 + 651.1

A HANDY GUIDE TO RATE OF INCREASE INTERPRETATIONS

Abstract. Compound interest tables are reduced to a single 3x5 card for field use. Examples are given to illustrate the table's use.

Introduction

How many years will it take a stand's volume to triple if its volume growth rate is 3 percent? If you said 37 years you're right. Probably you would have to look in a compound interest table for 3 percent, search out a $(1+p)^n$ value of 3 and read off the number of years.

What would you expect a stand to be worth in 10 years if it's worth \$100 now and grows at a rate of 4 percent per year, and if stumpage values increase at 3 percent per year? If you responded \$200, you're right. Probably you would have to look in a compound interest table for 10 years and 7 (3+4) percent to discover that V_o (beginning value) would be multiplied by $(1+p)^n$ of 2 yielding V_n (ending value) of \$200.

If you want to be able to give on the spot answers to questions like this quickly and easily then the table that follows is for you. In 1911, Howard Krinbill[1] constructed a table to answer just such questions. Here his table has been modernized; minor inconsistencies have been ironed out, limits for the table's use have been determined and it has been put on a small card which can be carried in your pocket.

How the Guide was Constructed

Compound interest relationships are expressed in a formula

$$V_n / V_o = (1+p)^n$$

[1] Krinbill, Howard R. 1911. Biltmore Timber Tables. 13 pp. Biltmore Forest School.

where V_n = Quantity or value at year n
V_o = Quantity or value at year o, the beginning of the period
p = Compound interest rate
n = The number of years in the period

Compound interest tables display the value V_n/V_o for different combinations of p and n. A given V_n/V_o ratio may be produced by many combinations of p and n. The product pxn is always the same for a given V_n/V_o ratio and it is this product that is called a compound interest factor.

For example, if we had a value for V_o of \$150 and a value for V_n of \$300 then the ratio $V_n/V_o = 2.0$. Other combinations of p and n may also produce this ratio but pxn (compound interest factor) is always the same.

V_n/V_o ratio	*p*	*n*	*pxn*
2.0	1	70	70
2.0	2	35	70
2.0	5	14	70
2.0	7	10	70
2.0	10	7	70

These principles are used to formulate the table of compound interest factors on the attached card.

The numbers down the left side and across the top are units and tenths of the V_n/V_o ratio: the ratio of the final value to the initial value of the quantity in which the user is interested. For example, a tree containing 100 fbm grows to 260 fbm in a period of n years. The ratio is $V_n/V_o = \frac{260}{100} = 2.6$. Reading across from 2 on the left side and down from 0.6 at the top, we would find a value of 96 at the intersection. The table value is the product of p, (the rate of increase) times n (the number of years). To determine p we divide the table value by n. To find n we divide the table value by p. If our example tree had shown this increase in 30 years the rate of increase would be 96/30 = 3.2 percent. If, on the other hand, a tree of this character would be expected to increase in volume at a rate of 4 percent, then we would expect to achieve an increase of 2.6 times or 260 fbm in 96/4 or 24 years.

How to Use the Guide to Solve Some Typical Problems

Nature of Problem	*Given*	*Wanted*	*Solution*
1. Estimate the rate of value increase of a stand worth \$100 ten years ago which is now worth \$150 (Other quantities such as volume or basal area could be used.)	V_o = \$100 V_n = \$150 n = 10	p	Find the factor for $\frac{150}{100} = 1.5$ in the table (ans. 41). Divide 41/10 = 4.1 percent rate of value increase.
2. If a stand measure (e.g. volume) increases at the rate of 3 percent, how long will it take for this measure to double?	p = 3 percent $\frac{V_n}{V_o} = 2.0$	n	Find the factor $\frac{V_n}{V_o} = 2$ (ans. 70). Divide 70/3 percent = 23 years.
3. If a timbered tract has a land value of \$100 which is expected to increase at 7 percent per year and has \$75 of standing timber expected to increase at 4.7 percent per year (a) what is the estimated value of the whole investment in 10 years and (b) what rate is earned on the total investment?	V_{o1} = \$100 V_{o2} = \$ 75 p_1 = 7.0 percent p_2 = 4.7 percent n = 10	a) V_{n3} (the end of period value for V_{o1} and V_{o2} combined. b) p_3 (the rate of interest earned by the combined land and timber investment).	7 percent x 10 years = 70, a factor for $V_{n1}/V_{o1} = 2$ ∴ value of land. $V_{n1} = 2 \times \$100 = \$200$. 4.7 percent x 10 years = 47, a factor for $V_{n2}/V_{o2} = 1.6$ ∴ value of timber $V_{n2} = 1.6 \times \$75 = \$120$. a) value $V_{n3} = \$200 + \$120 = \$320$. b) rate of total investment p_3. Determine $\frac{V_{n3}}{V_{o3}} = \frac{320}{175} = 1.8$. Look up factor (ans. 59). Divide by 10 and p_3 = 5.9 percent.

Compound interest factors (pxn) for given rates of increase (V_n/V_o)

Rate of increase	Tenths									
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Units										
1		10	19	26	34	41	47	53	59	65
2	70	75	79	84	88	92	96	100	104	107
3	110	114	117	120	123	126	129	131	135	137
4	139	142	144	147	149	152	154	157	159	161
5	163	165	168	170	172	174	176	178	179	180
6	182	183	185	186	187	189	191	193	195	197
7	199	200	202	203	205	206	207	208	209	210
8	211	213	214	215	216	217	218	219	220	221
9	222	223	224	226	227	228	229	231	232	233
10	234									

Conceived by Howard R. Krinbill, 1911; modernized by David P. Worley, 1974.

Examples

1. A forest is expected to increase in volume from 7,700 fbm to 12,300 fbm in 10 years. What is the growth percentage?

   12,300 ÷ 7,700 = 1.6 factor for 1.6 is 47 47 ÷ 10 = 4.7%

2. If an industrial forest must make 6 percent on its investment in standing timber, how much will the value of standing timber have to increase if left alone for 5 years? 10 years? 20 years?

5 yrs. 6x5 = factor of 30 V_n/V_o = 1.3 to 1.4 — 1.0 = increase 30-40%
10 yrs. 6x10 = factor of 60 V_n/V_o = 1.8 — 1.0 = increase 80%
20 yrs. 6x20 = factor of 120 V_n/V_o = 3.3 — 1.0 = increase 230%

Accuracy

The table has some accuracy limitations with which the user should be acquainted. It must be recognized that this estimating table is the result of considerable rounding off so answers obtained by using it lack precision. Tests of the table reveal that estimates of n (number of years) are correct within 1 year. An answer of 47 years then may be as high as 48 or as low as 46 years. Answers in terms of p are not uniformly precise. Their limits are shown below.

If answer is between	*It may be off as much as*
0—5%	0.1%
6—10%	0.5%
10—15%	1.0%
15—20%	2.0%
20% and over	4.0%

Since most answers in the forestry field are in the 0 to 10 percent bracket these limitations are not considered serious. The accuracy of V_o and V_n determinations can be estimated by rerunning the usual procedure to estimate the limits. For example, to estimate the range of V_n at a p of 25 percent, we would estimate for 25 percent plus and minus 4 percent, or 21 percent and 29 percent. This would give the possible range of V_n values.

Extrapolation

This is a factoring type table so the user is not limited to the V_n/V_o ratios shown. He can extend these data by using V_n/V_o multiples and adding the factors. For example: What is the factor for a V_n/V_o ratio of 20? V_n/V_o ratios of 2 and 10 equal 20 when multiplied and their factors add up to the factor for 20.

Ratios: V_n/V_o of 10 x V_n/V_o of 2 = 20

Factors: for 10 = 234
+ for 2 = 70
for 20 = 304

Had we factored 20 using 4x5 for the ratios our factor for 20 would have been 139 + 163 = 302. The differences are due to rounding off as mentioned earlier. The accuracy limits given earlier still hold for these extrapolated values.

Conclusions

If you take a couple of hours to become familiar with this table, you can estimate rate relationships with incredible speed and understanding. But resist the temptation to solve the complex compound interest problems that are theoretically possible with this table. Rounding off errors will propagate in complex situations and give you misleading answers.

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MANUSCRIPT RECEIVED FOR PUBLICATION 16 OCTOBER 1973